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Thrust Loss Due to Supersonic Mixing

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The maximum thrust developed by a device in which two streams mix in a parallel configuration at supersonic velocities is estimated. Total pressure profiles in a two-dimensional, compressible shear layer are calculated by assuming turbulent Prandtl and Lewis numbers of unity. As the convective Mach number M_c rises, the total pressure acquires a defect that becomes large for $M_c > 1$. For shear layers with equal freestream total pressures, an analytical relation for the defect vs M_c is found. The extent and magnitude of the defect agrees well with experimental data. The loss in total pressure is connected to the loss in thrust of a simplified model of a scramjet. The thrust loss is about 30% for $M_c = 2$ and 50% for $M_c = 3$. The trends are insensitive to details of the shear-layer velocity profile and to the ratios of freestream quantities. The role of turbulent-energy dissipation in the reduction of total pressure is discussed.

Nomenclature

a = speed of sound
C_T = thrust coefficient
c_p = specific heat at constant pressure
c_v = specific heat at constant volume
H = total enthalpy
h = static enthalpy
Le_t = turbulent Lewis number
M = Mach number
M_c = convective Mach number
Pr_t = turbulent Prandtl number
p = static pressure
p_T = total pressure
p'_T = pitot pressure
R = gas constant
s = entropy
T = static temperature
T_T = total temperature
$\mathcal{T}$ = thrust
U = freestream velocity
u = local mean velocity
x = streamwise coordinate
y = transverse coordinate
γ = specific-heat ratio
ΔU = $U_1 - U_2$
δ = shear-layer thickness
δ' = shear-layer growth rate
τ = shear stress

Subscripts
e = nozzle exit
1 = stream 1 (fast)
2 = stream 2 (slow)

Introduction

THE desire to generate thrust at hypersonic speeds using air-breathing propulsion has given rise to the concept of supersonic-combustion ramjet (scramjet) engines. From the practical point of view, two issues are central to the efficient operation of scramjets: 1) good fuel-to-air mixing and 2) minimization of total-pressure losses. The fact that airflow inside the combustion chamber is supersonic makes both goals extremely challenging. Although there is a multitude of ways one can inject gaseous fuel into air, they can broadly be classified into two categories: 1) transverse injection and 2) parallel injection. Transverse injection may produce good near-field mixing, but is inevitably accompanied by shocks that reduce the total pressure. Parallel injection, on the other hand, could conceivably be achieved without shocks, and therefore appears to be a more efficient way to mix, provided that the combustor is long enough to permit the desired amount of mixing. This view may be misleading, however, because at high Mach number viscous dissipation may result in losses comparable to losses suffered through a shock. Such losses would manifest themselves not just in parallel mixing, but in any kind of mixing where the characteristic velocity difference is on the order of the speed of sound or higher.

Issues related to pressure recovery and choking criteria in two-stream, high-speed mixing were addressed as early as the sixties.[1] However, there appear to be no systematic studies of total pressure in supersonic mixing. Since total pressure is directly linked to thrust, it is important to know the conditions under which serious losses can occur. This report analyzes the effect of Mach number on the total-pressure distribution in a parallel-mixing configuration, which is then connected to the thrust generated by a simplified model of a scramjet.

Flow Model

The fluid-mechanical quantity that best determines the ability of a device to produce thrust is the total pressure p_T. Here, we examine the effect of Mach number on the total-pressure distribution inside a plane shear layer composed of similar or dissimilar gases (Fig. 1). We take the flow to be fully developed turbulent, and consider only the mean (time-averaged) quantities.

For $Pr_t = 1$ and, in the case of dissimilar species $Le_t = 1$, $H = h + \frac{1}{2}u^2$ is a linear function of the mean velocity[2]:

$$H(y) = A + Bu(y) \tag{1}$$

where A and B are constants determined by the freestream conditions:

$$A = \frac{U_1H_2 - U_2H_1}{\Delta U}, \qquad B = \frac{H_1 - H_2}{\Delta U} \tag{2}$$

The total pressure can be written in terms of the total and static enthalpies:

$$\frac{p_T}{p} = \left(\frac{H}{h}\right)^{(\gamma/\gamma-1)} = \left(\frac{A + Bu}{A + Bu - u^2/2}\right)^{(\gamma/\gamma-1)} \tag{3}$$

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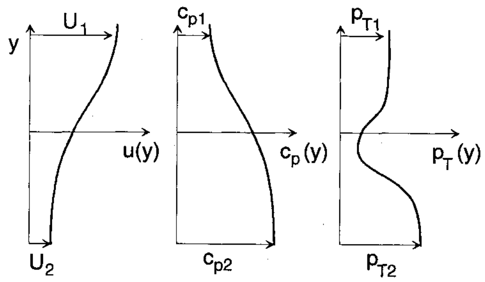

Fig. 1 Illustrative profiles for velocity, specific heat, and total pressure in a compressible shear layer.

The static temperature $T(y)$ and Mach number $M(y)$ are obtained from the adiabatic relation

$$\frac{H}{h} = \frac{T_T}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{4}$$

Whether the total-enthalpy difference is created by using gases with different total temperatures or different specific heats is immaterial to the determination of the above parameters.

For $\gamma_1 \neq \gamma_2$, the $Le_t = 1$ assumption implies that specific heats are linear functions of velocity and leads to the following expression for γ:

$$\gamma(y) = \frac{c_{p1}(u - U_2) - c_{p2}(U_1 - u)}{c_{v1}(u - U_2) + c_{v2}(U_1 - u)} \tag{5}$$

Unless the freestream γ are very different, the effect of γ variation on total pressure is minor.

An additional quantity of practical importance is the pressure p'_T measured by a pitot probe which, for supersonic Mach numbers, equals the pressure behind a normal shock and is given by the well-known Rayleigh formula

$$\frac{p'_T}{p} = \left[\frac{(\gamma + 1)M^2}{2}\right]^{(\gamma/\gamma - 1)} \left[\frac{\gamma + 1}{2\gamma M^2 - (\gamma - 1)}\right]^{(1/\gamma - 1)} \tag{6}$$

Intuitively, we expect that any change in the behavior of total pressure should be governed by the Mach number based on the velocity difference, rather than the freestream Mach numbers. After all, a shear layer with high freestream Mach numbers, but very small velocity difference, approaches the state of uniform flow with zero dissipation. A popular parameter that describes compressibility based on velocity difference is the convective Mach number[3,4]

$$M_c = \frac{\Delta U}{a_1 + a_2} \tag{7}$$

that we will use as a measure of the intrinsic compressibility of the shear layer.

To plot sample total-pressure distributions vs M_c, a hyperbolic-tangent velocity profile of the form

$$u(y) = \tfrac{1}{2}(U_1 + U_2) + \tfrac{1}{2}(U_1 - U_2)\tanh(\beta y) \tag{8}$$

is assumed, where β is a constant chosen to give the desired profile thickness. We consider a shear layer composed of hydrogen and air at the same total temperature. In Fig. 2, profiles of total pressure are plotted vs M_c for two cases, one with $p_{T_2} = p_{T_1}$ ($M_2 = M_1$), and the other with $p_{T_2} = 0.5p_{T_1}$ ($M_2 < M_1$). The transverse coordinate range, from -0.5 to 0.5, corresponds to a profile thickness defined from 1 to 99% of ΔU. The convective Mach number is increased by increasing M_1 and M_2. It is seen that M_c has a profound effect on the total pressure, which in both cases acquires a significant defect as M_c exceeds 1.

Analytical Expression for Total-Pressure Minimum

As seen in Fig. 2a, a defect in the total-pressure profile is especially apparent when the freestream total pressures are equal. It will now be shown that with $p_{T_1} = p_{T_2}$, the magnitude of the defect is explicitly related to the convective Mach number M_c. For simplicity, it is assumed that $\gamma_1 = \gamma_2$, which means that $M_1 = M_2$. In that case, the convective Mach number can be written as

$$M_c = M_1 \Delta U/(U_1 + U_2) \tag{9}$$

We now examine the existence of a minimum in the ratio of local-to-freestream total pressures

$$\frac{p_T}{p_{T_1}} = \left(\frac{H}{h}\frac{h_1}{H_1}\right)^{(\gamma/\gamma - 1)} \tag{10}$$

That minimum must coincide with the minimum of the ratio H/h. To locate it, we set

$$\frac{\mathrm{d}}{\mathrm{d}u}\left(\frac{H}{h}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(\frac{A + Bu}{A + Bu - u^2/2}\right) = 0$$

and find the velocity u_m at which the minimum occurs:

$$u_m = 2A/B \tag{11}$$

Clearly, if u_m falls outside the bounds (U_1, U_2), the profile does not have a true minimum, i.e., it is monotonic. The minimum value of H/h is

$$(H/h)_m = 1/(1 - u_m/B) \tag{12}$$

For $p_{T_1} = p_{T_2}$, from Eq. (10) the enthalpies must satisfy the relation

$$H_1/h_1 = H_2/h_2$$

and therefore the constants A and B in Eq. (1) must satisfy

$$A/B = -U_1U_2/(U_1 + U_2)$$

hence, Eq. (11) becomes

$$u_m = U_1U_2/U_{\text{avg}} \tag{13}$$

where $U_{\text{avg}} \equiv \frac{1}{2}(U_1 + U_2)$ is the average of the freestream velocities. By writing the static enthalpy in terms of the sound speed $h = a^2/(\gamma - 1)$, and doing the subtraction $h_2 - h_1 = B(U_2 - U_1) - \frac{1}{2}(U_1^2 - U_2^2)$, we find the following relation for B:

$$B = \underset{2}{U_{\text{avg}}}[1 + (1/k)] \tag{14}$$

where $k \equiv [(\gamma - 1)/2]M_1$. Substituting Eqs. (13) and (14) in Eq. (12) we have

$$\left(\frac{H}{h}\right)_m = \frac{1 + k}{1 + k[1 - (U_1U_2/U_{\text{avg}}^2)]}$$

$$\left(\frac{H}{h}\right)_m \frac{h_1}{H_1} = \frac{1 + k}{1 + k[1 - (U_1U_2/U_{\text{avg}}^2)]}\,\frac{1}{1 + k} \tag{15}$$

$$= \frac{1}{1 + [(\gamma - 1)/2]M_1^2[\Delta U/(U_1 + U_2)]^2}$$

Recognizing the product of the last two terms in the denominator as M_c^2 [see Eq. (9)], we substitute Eq. (15) in Eq. (10) and obtain

$$\frac{p_{Tm}}{p_{T_1}} = \left[1 + \frac{\gamma - 1}{2} M_c^2\right]^{-(\gamma/\gamma - 1)} \tag{16}$$

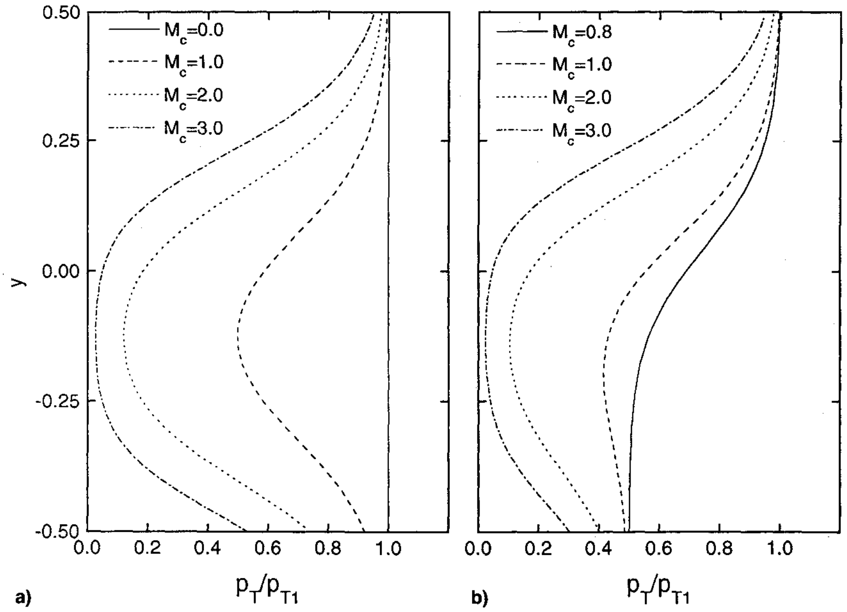

Fig. 2 Total-pressure profiles vs M_c for hydrogen-air shear layer with $T_{T_1} = T_{T_2}$: a) $p_{T_2} = p_{T_1}$ and b) $p_{T_2} = 0.5p_{T_1}$.

which links explicitly the total-pressure minimum, and thus the defect, to the convective Mach number. An obvious feature of Eq. (16) is that the magnitude of the defect does not depend on the shape of the velocity profile, as long as that profile is monotonic.

Comparison with Experiment

There are no experiments known to the author designed specifically for the study of total pressure in supersonic-mixing configurations. However, pitot surveys are often used to determine the thickness of the shear layer. One such survey from the work of Papamoschou and Roshko[4] was measured at conditions that highlight the total-pressure deficit, i.e., with $p_{T_1} \approx p_{T_2}$. Their shear layer was composed of helium ($\gamma = 5/3$) at $M_1 = 2.6$, and nitrogen ($\gamma = 1.4$) at $M_2 = 2.8$. The convective Mach number was $M_c = 1.1$, and the velocity ratio was $U_2/U_1 = 0.42$. The profile considered here was measured in the fully developed part of the shear layer and is well documented in Ref. 5.

To test the theoretical model, we compute a case with the same Mach numbers and gases as in the above experiment, assuming a velocity profile of the form given by Eq. (8). The total and pitot pressures are obtained from Eqs. (3) and (6), with $\gamma(y)$ given by Eq. (5). The constant β in Eq. (8) was adjusted so that the "pitot thickness" (see Ref. 4) matched the experimental one of 6.9 mm. The comparison between model and experiment is shown in Fig. 3. The model predicts a minimum pitot pressure $p'_{Tm}/p = 6.6$ compared to $p'_{Tm}/p = 6.0$ for the experiment. It slightly underpredicts the extent of the defect, probably because the experimental velocity profile is more linear than hyperbolic-tangent. Overall, the agreement is surprisingly good, considering the simplicity of the model.

Since the experimental freestream total pressures were roughly equal, we can use Eq. (16) to estimate the pitot-pressure minimum without computing the entire profile. With $M_c = 1.1$, $p_{T_1} \approx p_{T_2} \approx 23p$ and $\gamma = 1.53$ (the average of freestream γ), Eq. (16) gives $p_{Tm}/p = 10.2$. From that, we obtain $M_m = 2.15$ and $p'_{Tm}/p = 6.9$, which is close to the value of 6.6 predicted by the full model.

Implications for Hypersonic Thrust

To assess the practical consequences of the total-pressure deficit on hypersonic propulsion, we consider the device depicted in Fig. 4 in which two gases, supplied from two

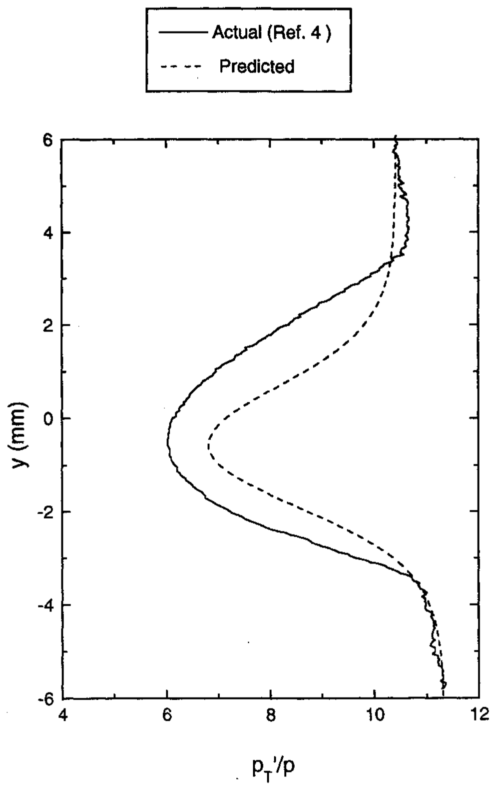

Fig. 3 Actual and predicted pitot-pressure profiles for shear layer composed of He at Mach 2.6 and N_2 at Mach 2.8.

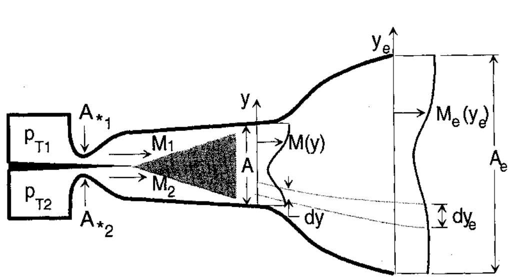

Fig. 4 Conceptual propulsion device.

independent reservoirs, pass through sonic areas and mix at compressible speeds in a two-dimensional shear-layer, constant-pressure configuration. When the shear-layer thickness (defined here from 5 to 95% of ΔU) occupies the entire channel height A, the mixture expands through a diverging nozzle of exit area A_e into vacuum. This is not a practical propulsion device but a conceptual one that allows us to evaluate the thrust in a systematic fashion and establish trends that will apply to a variety of parallel-mixing configurations.

For exhaust into vacuum, the thrust is

$$\mathcal{T} = p_e \int_{A_e} (1 + \gamma_e M_e^2)\, dy_e \tag{17}$$

The maximum thrust occurs when the exit area is very large ($A_e \to \infty$, $M_e \to \infty$), for which Eq. (17) is approximated by

$$\mathcal{T} = p_e \int_{A_e} \gamma_e M_e^2\, dy_e \tag{18}$$

To relate the conditions at the nozzle exit A_e to those at the entrance A, the nonuniform nozzle flow is approximated by a series of stream tubes with uniform conditions in each. Viscous effects are neglected and the flow within each stream tube is assumed isentropic and of constant concentration, i.e., $\gamma_e(y_e) = \gamma(y)$. The static pressure is uniform at the entrance and exit. This is similar to the heterogeneous-mixing analysis of Ferri and Edelman.[1] We rewrite Eq. (18) in the form

$$\mathcal{T} = \int_A M_e^2 \frac{p_e}{p_T} \frac{dy_e}{dy} \gamma p_T\, dy \tag{19}$$

so that integration is now performed over the entrance area A. The pressure ratio in Eq. (19) is a function of M_e

$$p_e/p_T = f(M_e)$$

where $f(M)$ is the isentropic pressure relation

$$f(M) \equiv \{1 + [(\gamma - 1)/2]M^2\}^{-(\gamma/\gamma - 1)}$$

The change in stream-tube area from dy to dy_e can be referred to a virtual sonic area dy^*

$$\frac{dy_e}{dy} = \frac{\dfrac{dy_e}{dy^*}}{\dfrac{dy}{dy^*}} = \frac{g(M_e)}{g(M)}$$

where

$$g(M) \equiv \frac{1}{M}\left[\frac{2}{\gamma - 1}\left(1 + \frac{\gamma - 1}{2} M^2\right)\right]^{[\gamma + 1/2(\gamma - 1)]}$$

is the isentropic area-Mach number relation. In terms of f and g, the thrust is

$$\mathcal{T} = \int_A M_e^2 f(M_e) \frac{g(M_e)}{g(M)} \gamma p_T\, dy$$

2

It is easy to show that as $A_e \to \infty$, the product $M_e f(M_e) g(M_e)$ reaches a constant

$$\lim_{A_e \to \infty} M_e^2 f(M_e) g(M_e) = \left(\frac{\gamma - 1}{\gamma + 1}\right)^{[\gamma + 1/2(\gamma - 1)]} \left(\frac{\gamma - 1}{2}\right)^{-(\gamma - 1/\gamma)} \equiv \frac{G(\gamma)}{\gamma}$$

hence the thrust becomes

$$\mathcal{T} = \int_A G(\gamma) \frac{p_T}{g(M)}\, dy \tag{20}$$

Eq. (20) relates the thrust directly to the total-pressure distribution $p_T(y)$, thus illustrating how a defect in total pressure results in loss of thrust.

To express the thrust in nondimensional form, we define the thrust coefficient

$$C_T = \frac{\mathcal{T}}{p_{T_1} A_{*1} + p_{T_2} A_{*2}} \tag{21}$$

where

$$A_{*1} = \frac{A/2}{g(M_1)}, \qquad A_{*2} = \frac{A/2}{g(M_2)}$$

The definition of C_T in Eq. (21) is similar to the usual definition $C_T = \mathcal{T}/(p_T A_*)$ for rockets with a single gas reservoir.[6] For $\gamma = 1.4$, the maximum C_T for a rocket in space is 1.81.

Figure 5 depicts C_T vs M_c for a mixture of hydrogen and air with $T_{T_1} = T_{T_2}$ and $M_1 = M_2$ (hence, $p_{T_1} = p_{T_2}$). The convective Mach number M_c is increased by increasing M_1 and M_2. For example, the $M_c = 2$ point corresponds to $M_1 = M_2 = 3.5$. Two shear-layer velocity profiles are considered, one hyperbolic-tangent and the other linear. It is seen that C_T declines with increasing M_c, the trend being rather insensitive to the velocity profile. The thrust loss is approximately 10% at $M_c = 1$, 30% at $M_c = 2$ and 45% at $M_c = 3$. The low-M_c values, where the freestream Mach numbers are subsonic, reflect a ramjet rather than a scramjet configuration. A throat, placed immediately downstream of area A in Fig. 4, is then needed to expand the gases to supersonic speeds in the exhaust nozzle. Inclusion of such throat does not alter the thrust relation derived above. This comment applies also to all subsequent figures.

The effect of employing very different γ is seen in Fig. 6, where a fictitious mixture of gases with molecular weights 2 and 29 at the same freestream Mach numbers and total temperatures is considered. The velocity profile is hyperbolic-tangent. The loss in thrust is moderately higher when the high-speed stream has large γ (36% at $M_c = 2$ vs about 30% for

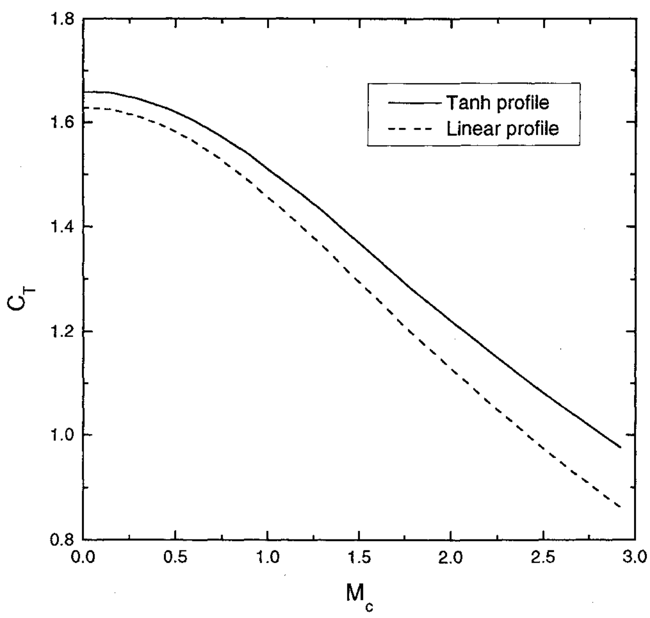

Fig. 5 Thrust coefficient vs M_c for hydrogen/air mixture with $M_1 = M_2$, $T_{T_1} = T_{T_2}$.

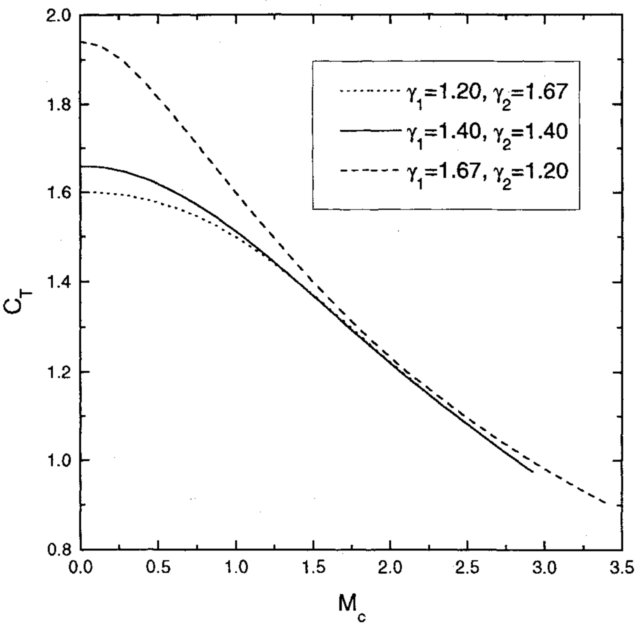

Fig. 6 Effect of γ on thrust coefficient for mixture of gases with $MW_1 = 2, MW_2 = 29, M_1 = M_2, T_{T_1} = T_{T_2}$.

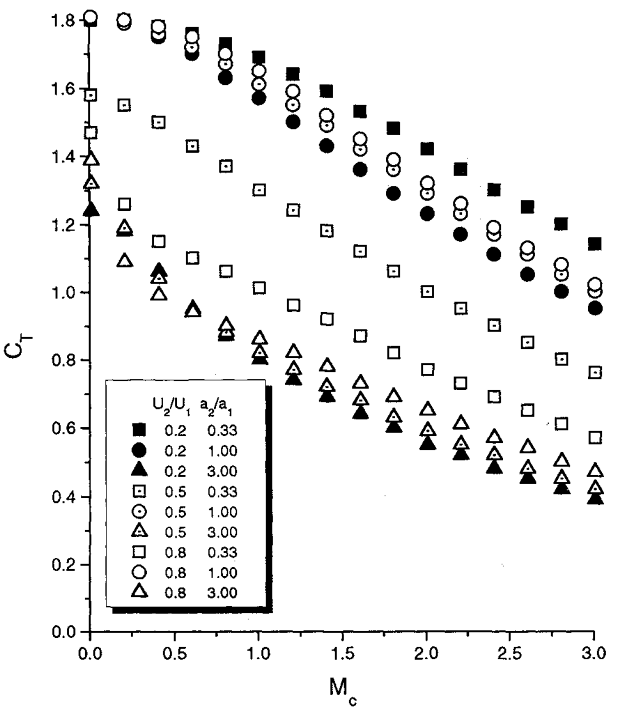

Fig. 7 Thrust coefficient vs M_c for various combinations of velocity and speed-of-sound ratios.

the two other cases). Since large variations in γ are unlikely to occur in an actual combustor, particularly if air and hydrogen are used, it is fair to say that the effect of γ on the thrust-vs-M_c relation is secondary.

To demonstrate that the above trends occur under any combination of freestream variables, Fig. 7 depicts C_T vs M_c for nine combinations of velocity and speed-of-sound ratios, with $0.2 \le U_2/U_1 \le 0.8$ and $0.33 \le a_2/a_1 \le 3.0$ ($\gamma_1 = \gamma_2$). All cases experience thrust loss as M_c increases. The decline of thrust in the subsonic-M_c range is faster for the cases with moderate-to-high U_2/U_1 and/or $a_2/a_1 \gtrsim 1$. It should be noted that the value $a_2/a_1 = 3$ is rather unrealistic since it implies that the heavy gas is faster, which is typically not the case. It is included here for completeness.

Discussion

It is significant that the only two assumptions made for the determination of the total-pressure profiles were 1) a monotonic velocity profile and 2) turbulent Prandtl number of unity and, in case of different species, turbulent Lewis number of unity. The first assumption is compatible with the definition of a shear layer, and we have seen that the exact profile shape plays no role on the magnitude of the defect. The second assumption implies that the turbulent exchange coefficients for momentum, heat, and mass are equal, which is a good approximation for gaseous flows.[7]

The loss in total pressure, and related loss in thrust, were arrived at by consideration of mean quantities only and without any knowledge of the salient turbulent mechanisms. It is also instructive to view the above findings from the perspective of turbulent-energy dissipation. We examine the equation for entropy s, which in its most general form is

$$\rho T \frac{Ds}{Dt} = \boldsymbol{T}:\nabla \boldsymbol{u} + \nabla \cdot \boldsymbol{q} \tag{22}$$

where $\boldsymbol{T}$ is the shear-stress tensor, $\boldsymbol{u}$ is the velocity vector, $\boldsymbol{q}$ is the heat transfer, and : denotes the inner product between two tensors. Focusing on the effect of the shear stress alone, and making the approximation of thin layer, the dominant term on the right side is

$$\rho T \frac{Ds}{Dt} = \tau \frac{\partial u}{\partial y} \tag{23}$$

For fully developed turbulent flow, τ is the Reynolds shear stress and the right side is proportional to the rate of dissipation of turbulent energy. In order-of-magnitude form, Eq. (23) is

$$\frac{1}{R}\frac{\Delta s}{\Delta t} = \frac{\tau}{\rho}\frac{\gamma}{a^2}\frac{\Delta U}{\delta} \tag{24}$$

where R has been assumed constant and the relation $a^2 = \gamma RT$ has been used.

Following Brown and Roshko's[8] argument, the shear stress can be expressed as

$$\tau = \alpha \delta' \rho U \Delta U \tag{25}$$

where $\delta' = d\delta/dx$ is the growth rate, U is on the order of the average freestream velocity, and α is a constant. Substituting in Eq. (24), we have

$$\frac{1}{R}\frac{\Delta s}{\Delta t} = 4\gamma\alpha \frac{\delta'}{\delta} U M_c^2 \tag{26}$$

where $M_c = \Delta U/(2a)$. The characteristic length traveled by a fluid particle from the freestream to the middle of the shear layer is $x = \delta/\delta'$, and the corresponding time is

$$\Delta t = \frac{x}{U} = \frac{\delta}{\delta' U}$$

Hence, the entropy change experienced by that particle is

$$\Delta s/R = 4\gamma\alpha M_c^2 \tag{27}$$

The constant α being on the order of 0.1,[9] the entropy change for $M_c = 1$ and $\gamma = 1.4$ is $\Delta s/R \approx 0.5$. The resulting total-pressure ratio is

$$p_T/p_{T_1} = e^{-\Delta s/R} \approx 0.6$$

which is consistent with the values obtained by the model (see Fig. 2a).

The present study does not encompass reacting flows because their analysis would be much more complex, requiring proper models for the sources of heat and new species. Nevertheless, it appears reasonable to expect that the dissipative mechanisms that cause large total-pressure drop in the high-M_c, nonreacting case will not be fundamentally altered by reaction. Hence, the trends seen here are very likely to be seen in a reacting flow, especially if they are correlated with a form of M_c that takes into account the potentially high values of sound speed inside the reaction zone.

Conclusions

The total-pressure distribution in a compressible shear layer is a function only of velocity, provided that $Pr_t = Le_t = 1$. The resulting algebraic expression for p_T reveals the occurrence of a substantial defect as the convective Mach number M_c becomes supersonic. For shear layers with equal freestream total pressures, an explicit relation for the defect M_c is obtained. Comparison with experimental data shows that the model predicts the defect to within 10%. The loss in total pressure is connected to the loss in thrust of a simplified model of a scramjet. The thrust loss is about 30% for $M_c = 2$ and 50% for $M_c = 3$. The trends are insensitive to details of the shear-layer velocity profile and to the freestream values of velocity, density, and specific-heat ratio. The total-pressure deficit, predicted solely on the basis of mean profiles, is consistent with order-of-magnitude estimates obtained by relating the entropy rise to the rate of dissipation of turbulent energy.

Acknowledgment

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